MATH3404 Week 6

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Finding minimising curves

$$J[x] = \int_{t_0}^{t_1} f(t, x, \dot{x})dt, \quad x(t_0) = x_0, \quad x(t_1) = x_1$$

We know how to find the extremels, but how can we show that an extremel is a minimising curve?

- Let $y(t) = x(t) + \eta(t)$, where $x(t)$ is an extremel and $\eta \in C^1$ with $\eta(t_0) = \eta(t_1) = 0$.
- Show that $J[y] - J[x] \geq 0$.
- Every C^1 curve passing through the endpoints is of the form $y(t) = x(t) + \eta(t)$, so x is a minimising curve.

Lagrange multipliers (again)

Problem

Find the extremels of

$$J[x] = \int_{t_0}^{t_1} f(t, x, \dot{x})dt \tag{1}$$

with $x(t_0) = x_0$, $x(t_1) = x_1$ *and subject to the constraint*

$$\int_{t_0}^{t_1} g(t, x, \dot{x})dt = c.$$

Theorem

Extremels of (1) *are also extremels of*

$$\int_{t_0}^{t_1} (f + \lambda g)dt,$$

where $\lambda \in \mathbb{R}$.