

# MATH3404 Week 12

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Slides at <https://simonnthomasmaths.github.io/teaching/>

# Systems with complex eigenvalues

## Problem

*The system*

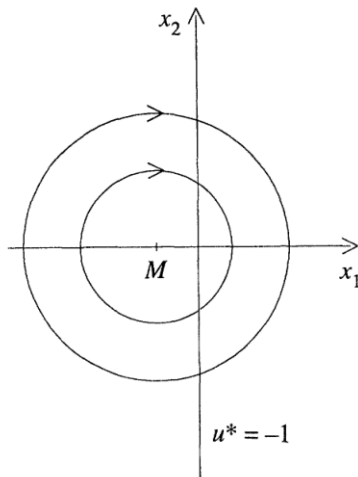
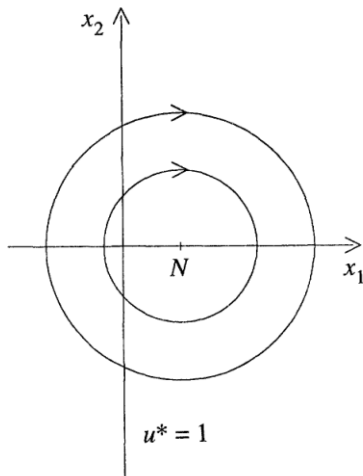
$$\dot{\mathbf{x}} = A\mathbf{x} + u\mathbf{l}$$

*is to be controlled to the origin in minimum time subject to  $|u| \leq 1$ .*

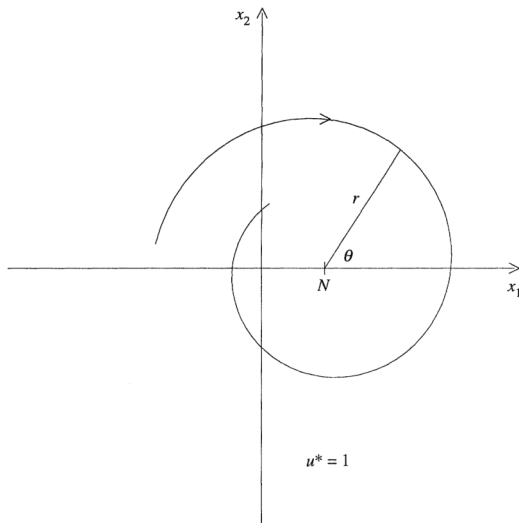
When  $A$  has real eigenvalues, we are able to find the optimal control and trajectories without finding  $\psi_1$  and  $\psi_2$ , since we knew that there can be at most one switch. When the eigenvalues are complex, we must find  $\psi_i$ .

Depending on the real part of the complex eigenvalues, we have three types of behaviour.

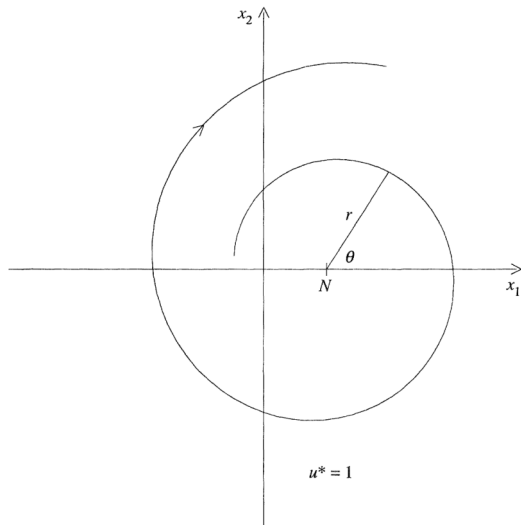
# Imaginary eigenvalues



# Eigenvalues with negative real part



# Eigenvalues with positive real part



## Problem

*Control the system*

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = -x_1 + u$$

*to the origin in minimum time with initial state  $(-2, 0)$  and constraint  $0 \leq u \leq 1$ . Calculate the minimum time.*